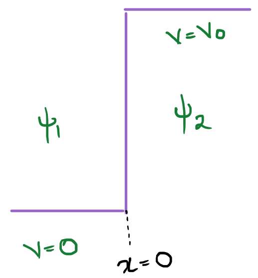
Tutorial Sheet-10

(1)



$E < V_0 \Rightarrow \psi_2(x) = Ce^{-\alpha x}$ where $\alpha = \sqrt{\dfrac{2m(V_0-E)}{\hbar^2}}$

$$\frac{P(x=x_0)}{P(x=0)} = \frac{c^2 e^{-2\alpha x_0}}{c^2} = \frac{1}{e} \Rightarrow e^{2\alpha x_0} = e$$

$$\Rightarrow \boxed{x_0 = 1/2\alpha}$$

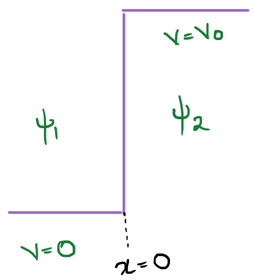
(b) $\Delta x = x_0 = 1/2\alpha \Rightarrow$ HUP $= \Delta x \Delta p \geq \frac{\hbar}{2} \Rightarrow \Delta p = \hbar\alpha = \sqrt{2m(V_0-E)}$

$$E = \frac{P^2}{2m} \Rightarrow \Delta E = \frac{2p\Delta p}{2m} = \frac{p}{m} \times \hbar\alpha = 2(V_0-E)$$

$$\Rightarrow E + \Delta E = 2V_0 - E \text{ and } E - \Delta E = 3E - 2V_0$$

$\Rightarrow$ E may exceed the value of V_0

(Q2)

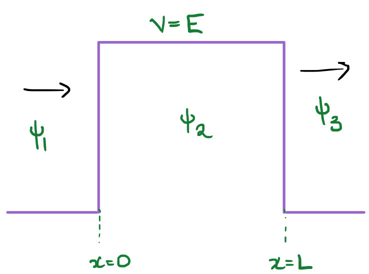


Same as 1a, with values.

$V_0 = 7eV,\ E = 3eV$

$$\frac{P(x=x_0)}{P(x=0)} = \frac{1}{2} \Rightarrow e^{-2\alpha x_0} = 1/2 \Rightarrow \boxed{x_0 = \frac{\ln 2}{2\alpha}}$$

(Q3) a)



$\psi_1 = \underline{Ae^{ik_1x}} + Be^{-ik_1x}$

$\psi_2 = Cx + D$ — because $V=E$!

$\psi_3 = Ee^{ikx}$

$(x=0)$

$$A+B=D \quad \text{— (1)}$$

$$(A-B)ik = C \Rightarrow A-B = \frac{-iC}{k} \quad \text{— (2)}$$

$(x=L)$

$$CL+D = Ee^{ikL} \quad \text{— (3)}$$

$$C = Eike^{ikL} \quad \text{— (4)}$$

$$(1)+(2) \Rightarrow 2A = D + \frac{C}{ik} \; ; \; (4)\rightarrow(3) \Rightarrow D = (Ee^{ikL})(1-ikL)$$

$$\Rightarrow 2A = (Ee^{ikL})(1-ikL) + \frac{Eike^{ikL}}{ik} = (Ee^{ikL})(1-ikL+1)$$

$$\Rightarrow \frac{E}{A} = \frac{2e^{-ikL}}{2-ikL} \Rightarrow \boxed{\text{Transmission Coeff.} = \left|\frac{2e^{-ikL}}{2-ikL}\right|^2 = \frac{4}{4+k^2L^2}}$$

(b) For Transmission $= 1/2$; $8 = 4+k^2L^2 \Rightarrow kL = 2 \Rightarrow \boxed{L = \lambda/\pi}$

(Q4) For the claim to be correct; the wavefunction needs to be continuous and differentiable.

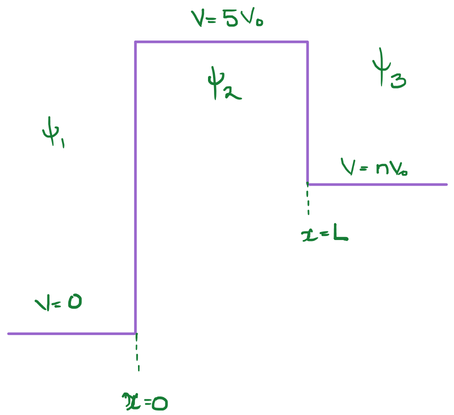
Cont. :- $Ae^{-k_1L} = Be^{-k_2L}$

Diff :- $-Ak_1e^{-k_1L} = -Bk_2e^{-k_2L}$

$\Rightarrow k_1 = k_2$ which is NOT true!

Therefore, the claim is false.

(Q5) * Correction in the question, $d = \dfrac{\pi\hbar}{\sqrt{8mV_0}}$ *



$$\psi_1 = Ae^{ik_1x} + Be^{-ik_1x} \qquad k_1 = \sqrt{\frac{2m(9V_0)}{\hbar}}$$

$$\psi_2 = Ce^{ik_2x} + De^{-ik_2x} \qquad k_2 = \sqrt{\frac{2m(4V_0)}{\hbar}}$$

$$\psi_3 = Ee^{ik_3x} \qquad k_3 = \sqrt{\frac{2m(9-n)V_0}{\hbar}}$$

$(x=0)$

$$A+B = C+D \quad — ①$$

$$k_1(A-B) = k_2(C-D) \quad — ②$$

$(x=L)$

$$Ce^{ik_2d} + De^{-ik_2d} = Ee^{ik_3d} \quad —— ③$$

$$k_2\left(Ce^{ik_2d} - De^{-ik_2d}\right) = k_3Ee^{ik_3d} \quad —— ④$$

** But $d = \dfrac{\pi\hbar}{\sqrt{8mV_0}} \Rightarrow k_2d = \pi \Rightarrow \left(e^{\pm ik_2d} = 1\right)^{**}$ d is needed here!

$$\Rightarrow \quad A+B = C+D, \qquad C+D = Ee^{ik_3d}$$

$$k_1(A-B) = k_2(C-D) \qquad k_2(C-D) = k_3Ee^{ik_3d}$$

$$\frac{A+B}{A-B} = \frac{k_1}{k_2}\left(\frac{C+D}{C-D}\right), \qquad \frac{C+D}{C-D} = \frac{k_2}{k_3}$$

Transmission = $3/4 \Rightarrow$ Ref = $1/4$

$$\Rightarrow \quad A/B = \pm 2$$

$$\Rightarrow \left(\frac{A+B}{A-B}\right)\left(\frac{2}{3}\right) = \frac{2}{\sqrt{9-n}}$$

$$\Rightarrow 9-n = 81 \text{ or } 1$$

$$\boxed{n = -72 \text{ or } 8}$$

(b) For $n = 8$; $\frac{C+D}{C-D} = \frac{2}{1}$; $\frac{B}{A} = \frac{1}{2}$ $\Rightarrow$ $C = \frac{9A}{8}$ and $D = \frac{3A}{8}$

(c) $B = A\left(\frac{k_1 - k_3}{k_1 + k_3}\right) \Rightarrow$ Phase change $= Im\left(\frac{k_1 - k_3}{k_1 + k_3}\right)$

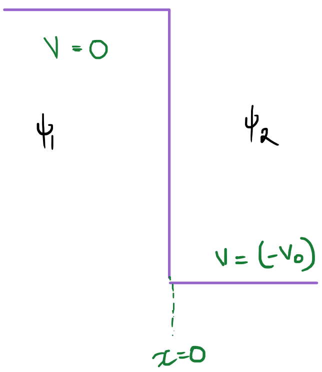
$$= Im\left(\frac{3 - \sqrt{9-n}}{3 + \sqrt{9-n}}\right) = 0$$

at $\underline{n=9}$

$\neq 0$

at $\underline{\underline{n = 72}}$

6)



$$\psi_1(x) = Ae^{ikx} + Be^{-ikx}$$

$$\psi_2(x) = Ce^{ik'x}$$

$$k = \frac{\sqrt{2mE'}}{\hbar} \qquad E' = E\cos^2\theta$$

$$k' = \frac{\sqrt{2m(E'+V_0)}}{\hbar}$$

$$\Rightarrow A + B = C \quad \text{—} ①$$

$$Ak - Bk = Ck' \quad \text{—} ②$$

Dividing,

$$\frac{A+B}{A-B} = \frac{k}{k'} \Rightarrow Ak' + Bk' = Ak - Bk$$

$$A(k'-k) = -B(k'+k)$$

$$\frac{B}{A} = \frac{k-k'}{k+k'}$$

Reflective Coefficient :- $\frac{|B|^2}{|A|^2} = \left|\frac{k-k'}{k+k'}\right|^2 = \left(\frac{\sqrt{E'+V_0} - \sqrt{E'}}{\sqrt{E'+V_0} + \sqrt{E'}}\right)^2 = \frac{(\sqrt{E'+V_0} - \sqrt{E'})^4}{V_0^2}$

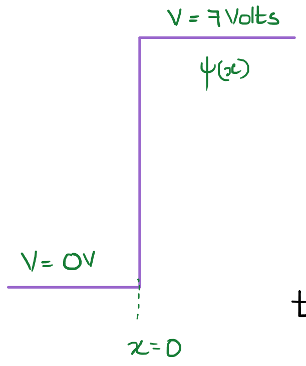
$E = \frac{p^2}{2m} \Rightarrow p = \sqrt{2mE} \Rightarrow p_\perp = \sqrt{2mE}\cos\theta$

$$\Rightarrow P_{ref} = P_\perp \cdot K \Rightarrow \text{fraction} = \frac{P_\perp \cdot K}{P} = K\cos\theta$$

$$= \frac{(\sqrt{E'+V_0} - \sqrt{E'})^4 \cos\theta}{V_0^2}$$

Q7) Work function = 7eV ⇒ barrier potential = 7 V

⇒ V_0 = 7 Volts.



$$\psi(x) = Ae^{-\alpha x} \quad \text{where} \quad \alpha = \frac{\sqrt{2m(V_0 - E)}}{\hbar} = \frac{2\sqrt{me}}{\hbar}$$

tunneling current at $x = 3A°$ ⇒ $|\psi(3\times10^{-10})|^2$

$$\Rightarrow A^2 e^{-2\alpha d}$$

" at $x = 6A°$ ⇒ $|\psi(6\times10^{-10})|^2$

$$\Rightarrow A^2 e^{-4\alpha d}$$

$$\text{Amplification} = 10\log_{10}\left(e^{2\alpha d}\right) = 7.87\times10^{-9}\ \underline{\underline{\text{dB}}}$$