

Mini Quiz 2

$$a) \theta = \pi/3 \Rightarrow \Delta\lambda = \lambda_c(1 - \cos\pi/3) = \lambda_c(1 - 1/2) = \lambda_c/2$$

$$b) \frac{3\lambda}{4} - \lambda' = \frac{\lambda_c}{2} \Rightarrow \lambda' = \frac{3\lambda_c}{4} - \frac{\lambda_c}{2} = \frac{\lambda_c}{4}$$

Mini Quiz 3

$$h\nu = kT \text{ — known.}$$

Model vibrations as 

$$\Rightarrow \nu = \frac{kT}{h}$$

$$\sqrt{\frac{k}{m}} = \frac{kT}{h} \Rightarrow T \propto m^{1/2}$$

$$\Rightarrow m_2 = 207 \left(\frac{8}{16}\right)^2 //$$

Mini Quiz 4

$$v_p = \omega/k = Ak^{1/2}$$

$$v_g = \frac{d\omega}{dk} = \frac{3}{2}Ak^{1/2}$$

Mini Quiz 5

$$g(\omega) = \frac{1}{\sqrt{2\pi}} \int f(t) e^{-i\omega t} dt = \frac{1}{\sqrt{2\pi}} \int_{-t_0}^{t_0} A e^{-i\omega t} dt = \frac{A}{\sqrt{2\pi}} \left[\frac{e^{-i\omega t}}{-i\omega} \right]_{-t_0}^{t_0}$$

$$= \frac{A}{\sqrt{2\pi}} \left[\frac{e^{i\omega t_0} - e^{-i\omega t_0}}{i\omega} \right] = \frac{A}{\sqrt{2\pi}} \left[\frac{2i \sin(\omega t_0)}{i\omega} \right]$$

$$= \frac{2A \sin(\omega t_0)}{\omega \sqrt{2\pi}}$$

Mini Quiz-6

$$\begin{aligned}\psi(x) &= A \sin\left(\frac{n\pi x}{L}\right) \Rightarrow \int \psi \psi^* dx = 1 \\ &\int_0^L A^2 \sin^2\left(\frac{n\pi x}{L}\right) dx = 1 \\ &\Rightarrow A^2 \int_0^L \frac{1 - \cos\left(\frac{2n\pi x}{L}\right)}{2} dx = 1 \\ &\Rightarrow A^2 \left[\frac{x}{2} - \frac{\cos\left(\frac{2n\pi x}{L}\right)}{2} \right]_0^L = 1 \Rightarrow A = \sqrt{\frac{2}{L}} e^{i\theta} \rightarrow \text{not needed, but eh...}\end{aligned}$$

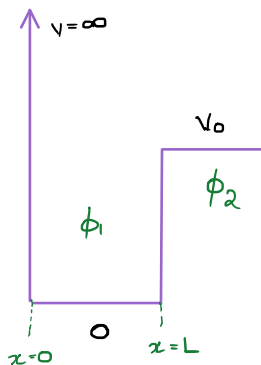
Mini Quiz-7

$$\psi(x) = A \sin\left(\frac{5\pi x}{2L}\right) \cos\left(\frac{\pi x}{2L}\right) = \frac{A}{2} \underbrace{\sin\left(\frac{3\pi x}{L}\right)}_{\psi_1} + \frac{A}{2} \underbrace{\sin\left(\frac{7\pi x}{L}\right)}_{\psi_2}$$

$$k^2 = \frac{2mE}{\hbar^2} \Rightarrow E = \frac{\hbar^2 k^2}{2m} \Rightarrow \langle E \rangle = \left[\left(\frac{3\pi}{L}\right)^2 \frac{\hbar^2}{2m} + \left(\frac{7\pi}{L}\right)^2 \frac{\hbar^2}{2m} \right] \left(\frac{1}{2}\right)$$

$$\Rightarrow E = \frac{13\pi^2 \hbar^2}{4mL^2}$$

Mini Quiz-8



$$\phi_1(x) = A \sin(kx) + B \cos(kx)$$

$$\phi_2(x) = C e^{-\alpha x}$$

$$\Rightarrow B=0 \text{ by continuity at } x=0 \Rightarrow \phi_1(x) = A \sin(kx)$$

$$(x=L) \Rightarrow A \sin(kL) = C e^{-\alpha L}$$

$$A k \cos(kL) = -C \alpha e^{-\alpha L}$$

$$\Rightarrow \tan(kL) = -k/\alpha$$

$$\tan(KL) = \frac{-K}{\alpha} \quad \text{where} \quad K = \sqrt{\frac{2mE}{\hbar^2}} = \sqrt{\frac{mv_0}{4\hbar^2}} = \frac{\sqrt{mv_0}}{2\hbar} //$$

$$\alpha = \sqrt{\frac{2m(v_0 - E)}{\hbar^2}} = \frac{\sqrt{7mv_0}}{2\hbar} //$$

$$\tan\left(\frac{\sqrt{mE}}{2\hbar} L\right) = \frac{-1}{\sqrt{7}} \Rightarrow \frac{\sqrt{mE}}{2\hbar} L = -\tan^{-1}\left(\frac{1}{\sqrt{7}}\right)$$

$$E = \frac{4\hbar^2}{mL^2} \left[\tan^{-1}\left(\frac{1}{\sqrt{7}}\right) \right]^2$$